

The Continuous Model of Stochastic *Mudharabah* Investment

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ABSTRACT

In Islam, all decisions, activities, policies, strategies and interactions in the economy are related to human relationships. In the Islamic financial system, the *syariah* rules are considered in all economic activities including investment. Investment is money or capital commitment for the purchase of financial instruments or other assets to gain benefits in the form of interest. Most investment opportunities are interest based but Islamic law strictly prohibits interest or *usury*, also called *riba* in Arabic. The prohibition of *riba* has led to the creation of alternatives schemes for the compensation of investment capital. One of the methods of compensation is by means of profit-sharing and one of the financial contracts that internalise profit sharing is *mudharabah*. It is an investment partnership in which one party called *rab ul mal* provides capital while the other party called *mudharib* brings labour and effort with the provision of profit sharing in some pre-determined proportions. This paper uses the model of stochastic *mudharabah* investment which can be used in forecasting the profits gain by both two parties in a stock market investment.

Keywords: Investment, mudharabah, stochastic

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INTRODUCTION

Islam is a complete code of life that is constructed upon the instructions given by Allah SWT and practices of the holy Prophet Muhammad SAW. In Islam, all decisions, activities, policies, strategies and interactions in the economy directly relates to the Hereafter because Islam makes no distinction between the spiritual

and secular world. Therefore, *syariah* rules must be considered in all economic activities, including investment.

Generally, investment relates to interest rates whenever money is invested. If the investment is responsive to interest rates, a small decrease in interest rates will lead to a considerable increase in investment (Lestari, 2010). Interest is an amount paid or received on top of the principal amount based on an agreement between two parties (Ahmad & Humayoun, 2011).

Most investment opportunities are interest based but Islamic law strictly prohibits interest (*usury*) commonly known as *riba* in Arabic (Akram, Rafique, & Alam, 2011). The prohibition against *riba* is found in several verses of the Holy Qur'an and the *Sunnah*. Most Islamic scholars define *riba* as a trade between the goods of the same types but in different quantities, where the increase would not be a proper compensation. According to *syariah*, *riba* is the premium to be paid by the borrower to the lender along with the principal amount as a condition for loans or to extend the loan period (Mohammed, 2009).

The prohibition against *riba* has been discussed by many Islamic scholars. In economic terms, there are those who say that the existence of *riba* will result in uneven distribution of wealth throughout the community by providing a vehicle for the rich to get richer and the poor to be poorer. Others believe that the modern

economic system has not supplied any explanation for the existence of interest rate requirement (Akram et al., 2011).

Alternative schemes for the compensation of investment capital have been introduced to address the problem of *riba*. The established method of compensation is by means of a profit and loss sharing (PLS) system. The nature and rationale of a PLS system is generally based on its essential features of total rejection of *riba* and establishing a financial alternative system free from this element.

Two types of financial contracts are offered in Islamic jurisprudence for substituting *riba* oriented transactions: *musyarakah* (partnership) and *mudharabah* (profit-sharing). *Musyarakah* as an investment partnership in which profit sharing terms are agreed upon in advance and losses are pegged to the amount invested. In the loss sharing concepts, all the partners in a joint business have to contribute funding and have the right but not the executive authority to implement the project (Ibrahim, Eng, & Parsa, 2009). *Mudharabah*, on the other hand, is an investment partnership in which one party called *rab ul mal* provides capital while the other party called *mudharib* brings labour and effort with the provision of profit sharing in some pre-determined proportions (Siddiqui, 2010). Table 1 shows the difference between *musyarakah* and *mudharabah* (Jaffar, 2006).

Table 1
The Difference between Mudharabah and Musyarakah

Mudharabah	Musyarakah
The capital is from one party (<i>rab ul mal</i>) and the effort is from other party (<i>mudharib</i>)	The capital is from all involved parties in accordance with an agreed number of shares
Investors are not allowed to work with entrepreneurs	Investors can carry out work in the company and can be rewarded
Losses incurred by investors	Losses incurred by all investors and entrepreneurs
Capital must be in the form of cash	Any property that may be valued in money can be used as a capital

The applications of these two PLS financings are mostly in the Islamic banking system. The Islamic banking and finance movement is a rapidly growing phenomenon in the Muslim world. Most conventional systems are based on profit and loss or the risk is shared by the parties in the contract. In conventional banking, the risks are borne by entrepreneurs whether the project is successful and produces profit or fails and results in losses, while capital owners will earn profits that have been predetermined. In Islam, this kind of unfair distribution is not allowed. In Islamic banking, both investors and entrepreneurs share outcome of the projects in a fair way. In the case of profit, both parties share them in pre-agreed proportions. All financial losses are borne by the capitalist and the entrepreneur. In order to manage unexpected losses from interest-based arrangements, orthodox Islamic finance argues that financing should be based on fair distribution of profit-loss

and risks. The Islamic investment model takes the form of either *mudharabah* or *musyarakah* (Farooq, 2007).

Islamic banking offers a PLS system in non-risky activities such as deposit accounts. The objective of this paper is to propose a new mathematical model for *mudharabah* and to apply it in high risk investment activities such as the stock market. The stock market is the place where shares in a company are bought and sold (Sincere, 2004). This gives the company option to access capital while giving the investors the opportunity to own shares in the company and enjoy the potential benefits of the company's future performance. The stock market is an indicator of the financial health of an economy. It indicates the mood of investors in a country (Tachiwou, 2010). Investors can only invest in shares through the stock exchange; an organised market in which shares are bought and sold under strict rules, regulations and guidelines.

This paper examines the *mudharabah* model using stochastic calculus. Stochastic calculus is very important in mathematical modelling of financial processes because of the underlying random nature of financial markets (Wilmott, 2007). The role of Brownian motion in stochastic processes is similar to that of normal random variables in elementary statistics. In random walk, the discrete counterpart of the (continuous time) Brownian motion is well-known among economics experts since most macroeconomics time series behave in a similar trend (random walk is a special case of what is known as a unit root process).

LITERATURE REVIEW

In the Islamic banking system, the method of financing is adjusted in accordance with Islamic principles. However, the current mathematical model of financing uses conventional methods which must be aligned to or consistent with Islamic principles or this can cast doubt on Islamic banking products. There is a dearth of research on mathematical models pertaining to Islamic banking activities.

The model presented by Khan (1986) is concerned with the overall principle of Islamic banks and did not discuss about specific products such as *mudharabah*. The model by Shaharir (1989) is based on the *mudharabah* principle (profit-sharing).

Presley and Sessions (1994) propose a western model of the *mudharabah* concept. They show that the use of *mudharabah* contract may be productive. The basic idea is that if the results of the project are stochastic and if the manager has more information about this stochastic over investors, then the *mudharabah* contract between both will lead a more efficient revelation of such information.

The models proposed by Jemain (1992) and Malek (2000) are on *musyarakah mutanaqisah* for purchase of commodities such as houses; the principle is to borrow to buy a commodity in the form of shares and pay the rent for these commodities. Jemain's mathematical model Jemain (1992) is identical with the common model if the ratio of rental price is the same with the current interest rate. The model presented by Malek (2000) is an

extension of Jemain's. This study focuses on the acquisition of the equity model of the borrower and the bank based on the principle of *musyarakah mutanaqisah*.

The models proposed by (Jaffar, 2006; Maheran, 2010) is on the new *musyarakah* model and *musyarakah mutanaqisah*, which takes into account the investment of two parties, the rate of profit, as well as two profit sharing rates.

The different models of *musyarakah* and *mudharabah* are discussed Jaffar (2010) and some applications are presented in Jaffar, Maad, Ismail and Samson (2012).

METHODOLOGY

In order to understand the derivative of the different equations, the *mudharabah* investment model introduced by Jaffar (2006) and Maheran (2010) are studied. The *mudharabah* investment partnership model is introduced as the equity of the capital provider, time t is E_t and the equity of the entrepreneur, time t is Q_t . The capital provider provides the initial capital and the entrepreneur manages the capital. The profit will be shared between the capital provider and the entrepreneur with a ratio of $k : (1 - k)$. At initial time t_0 there is no initial capital made by the entrepreneur but his equity can exist in this investment. The profit rate at time t is r_t . The *mudharabah* model can be mathematically expressed as follows:

$$E_t = E_{t-1} + r_t k E_{t-1} \text{ for } t = 1, 2, 3, \dots \quad (1)$$

and

$$Q_t = Q_{t-1} + r_t (1 - k) E_{t-1} \text{ for } t = 1, 2, 3, \dots \quad (2)$$

The model is to manage the deterministic and the non-risky type of investment.

In this research, the joint venture begins when the capital provider invests in risky businesses such as the stock market that moves randomly. The assumptions for the *mudharabah* investment model are as follows:

- the investment is between two parties (the capital provider and the entrepreneur).
- investing in one stock by using one volatility and one drift.
- investing in *syariah* counters only.
- parameter $k : (1 - k)$ is between 0 and 1. It can be in the form of a percentage or decimal point.
- the *mudharabah* investment model only considers the value of k when r_t is positive, $r_t > 0$
- assume there are no external factors such as seasonal factors, political issues, announcement from the company, natural disaster and etc.
- assume the *syariah* stock prices follow the log normal random walk.
- assume the *mudharabah* investment model follows the log normal random walk.

Equations (1) and (2) may be written in a matrix form,

$$\begin{aligned} \begin{pmatrix} E_t \\ Q_t \end{pmatrix} &= \begin{pmatrix} E_{t-1} \\ Q_{t-1} \end{pmatrix} + \begin{pmatrix} r_t k E_{t-1} \\ r_t (1 - k) E_{t-1} \end{pmatrix} \\ &= \begin{pmatrix} E_{t-1} \\ Q_{t-1} \end{pmatrix} + \begin{pmatrix} r_t k & 0 \\ r_t (1 - k) & 0 \end{pmatrix} \begin{pmatrix} E_{t-1} \\ Q_{t-1} \end{pmatrix} \\ \begin{pmatrix} E_t \\ Q_t \end{pmatrix} - \begin{pmatrix} E_{t-1} \\ Q_{t-1} \end{pmatrix} &= \begin{pmatrix} r_t k & 0 \\ r_t (1 - k) & 0 \end{pmatrix} \begin{pmatrix} E_{t-1} \\ Q_{t-1} \end{pmatrix} \end{aligned} \tag{3}$$

This equation can be further expressed as follows

$$X_t = X_{t-1} + \mathbf{A}_t X_{t-1} \quad t = 1, 2, 3, \dots \tag{4}$$

where $X_t = \begin{pmatrix} E_t \\ Q_t \end{pmatrix}$ and $\mathbf{A}_t = \begin{pmatrix} r_t k & 0 \\ r_t (1 - k) & 0 \end{pmatrix}$.

At the beginning of the period $t = 0$, the equity parts of capital provider and entrepreneur are E_0 and Q_0 respectively. If the rate of profit r_t is fixed which is r therefore matrix \mathbf{A}_t is scalar \mathbf{M} . The equation (4) becomes

$$X_t = X_{t-1} + \mathbf{M} X_{t-1} \quad \text{for } t = 1, 2, 3, \dots$$

where

$$X_0 = \begin{pmatrix} E_0 \\ Q_0 \end{pmatrix} \text{ and } \mathbf{M} = \begin{pmatrix} rk & 0 \\ r(1 - k) & 0 \end{pmatrix}.$$

The solution of this equation is by using matrix theory. The eigenvalues of matrix \mathbf{M} is rk and 0 . The *mudharabah* model has the profit-sharing rate k which satisfies $0 < k < 1$. The conclusion is that $\lambda_1 \neq \lambda_2$ and the matrix \mathbf{M} has distinct eigenvalues.

The Continuous Model of Mudharabah Investment

The continuous model for the *mudharabah* investment can be obtained from (3) by revealing it as (4).

The time unit is small enough such that the nano-seconds $t \rightarrow \infty$ until $X_t - X_{t-1} \rightarrow \dot{X}(t)$ which gives

$$\dot{X}(t) = \mathbf{A}(t)X(t) \tag{5}$$

with

$$\dot{X}(t) = \begin{pmatrix} \dot{E}(t) \\ \dot{Q}(t) \end{pmatrix}, X(t) = \begin{pmatrix} E(t) \\ Q(t) \end{pmatrix}$$

and

$$\mathbf{A}(t) = \begin{pmatrix} r(t)k & 0 \\ r(t)(1-k) & 0 \end{pmatrix}.$$

This equation is the same with the conventional investment model if $X(t)$ is the investment and $\mathbf{A}(t)$ is the profit rate at time t . The difference is that the conventional model is in scalar but the *mudharabah* model is in the vector. The significant difference is the conventional investment model involves only one party but the *mudharabah* model involves two parties that are capital provider and entrepreneur. Solving linear equations (5) gives us

$$X(t) = X(0)e^{\int_0^t A(S)dS}.$$

In the case $r(t) = \rho$ constant, the solution is $X(t) = X(0)e^{\mathbf{N}t}$ where \mathbf{N} is a matrix $\begin{bmatrix} \rho k & 0 \\ \rho(1-k) & 0 \end{bmatrix}$. In order to get

the values of $e^{\mathbf{N}t}$ as detailed, we must find the eigenvalues of matrix \mathbf{N} first. The eigenvalues of the matrix \mathbf{N} are $\lambda_1 = \rho k$ $\lambda_2 = 0$.

Similar to matrix \mathbf{M} , matrix \mathbf{N} also have the different eigenvalues which is $\lambda_1 \neq \lambda_2$. Hence, the following results:

$$e^{\mathbf{N}t} = \begin{bmatrix} (e^{\lambda_1 t})^2 - e^{\lambda_1 t} e^{\lambda_2 t} & 0 \\ (1-k) \left[\frac{e^{\lambda_2 t} - e^{\lambda_1 t}}{k} \right] & -e^{\lambda_1 t} \end{bmatrix} \tag{6}$$

The detailed solution for the case $r(t) = \rho$ is

$$\begin{pmatrix} E(t) \\ Q(t) \end{pmatrix} = \begin{bmatrix} (e^{\rho k t})^2 - e^{\rho k t} & 0 \\ (1-k) \left[\frac{1 - e^{\rho k t}}{k} \right] & -e^{\rho k t} \end{bmatrix} \begin{pmatrix} E(0) \\ Q(0) \end{pmatrix} \tag{7}$$

This equation allows us to calculate in detail the value of investment for both parties at any time and with the initial value of the investment.

The Continuous Model of Stochastic Mudharabah Investment

Let say in a joint venture with the concept of *mudharabah*, the investors want to invest in a risky investment such as in the stock market that the stocks are recognized by *syariah* on Bursa Malaysia. The initial capital for the capital provider and the entrepreneur are $E(0)$ and $Q(0)$.

The profits are shared with the ratio $k : (1-k)$ between a capital provider and

the entrepreneur respectively. Let say the equity of capital provider and the equity of entrepreneur at time t are $E(t)$ and $Q(t)$. The quantity $r(t)$ is the profit rate; the ratio $k:(1-k)$ is the rate of profit for capital provider and entrepreneurs.

If the investment at time t is without risk, the investment model is similar with equation (5). The nature of the development of stock prices is not fully known and it depends on the effects of random environment. This effect can be included in the rates of return/profit of $r(t)$.

Therefore,

$r(t) = \alpha(t) +$ a function of white noise.

$$r(t) = \alpha(t) + f(p) \tag{8}$$

where p is the white noise that involves unsystematic risk and systematic risk. The habits of white noise are not precisely known but it has distribution. A variable $\alpha(t)$ is a deterministic profit rate or profit rate that is identified for a risk-free investment. Substitute (8) into equation (5) will produce

$$\begin{aligned} \dot{E}(t) &= r(t)kE(t) \\ &= (\alpha(t) + f(p))kE(t) \\ &= \alpha(t)kE(t) + f(p)kE(t) \end{aligned}$$

and

$$\begin{aligned} \dot{Q}(t) &= r(t)(1-k)E(t) \\ &= (\alpha(t) + f(p))(1-k)E(t) \\ &= \alpha(t)(1-k)E(t) + f(p)(1-k)E(t) \end{aligned}$$

In this regard, the use of white noise as the standard is agreed and similar to the Brownian motion or Weiner process. Variable of $f(p)$ can be modelled as

$$f(p) = \lambda \dot{W}(t)$$

with $\dot{W}(t)$ as the standard white noise or Weiner process and λ is a constant. By using this in the previous two equations, we get

$$\dot{E}(t) = \alpha(t)kE(t) + \lambda \dot{W}(t)kE(t)$$

and

$$\dot{Q}(t) = \alpha(t)(1-k)E(t) + \lambda \dot{W}(t)(1-k)E(t).$$

This can be expressed as

$$\dot{X}(t) = \mathbf{C}(t)X(t) + \mathbf{D} \dot{W}(t)X(t) \tag{9}$$

where

$$\mathbf{C}(t) = \begin{pmatrix} \alpha(t)k & 0 \\ \alpha(t)(1-k) & 0 \end{pmatrix} = \alpha(t) \begin{pmatrix} k & 0 \\ 1-k & 0 \end{pmatrix}$$

is the expected drift rate of the investment risk of $X(t)$, while

$$\mathbf{D} = \lambda \begin{pmatrix} k & 0 \\ 1-k & 0 \end{pmatrix}$$

is the volatility of stock price $X(t)$. Equation (9) is in the form of differential equations that can be expressed as follows

$$\frac{dX(t)}{dt} = \mathbf{C}(t)X(t) + \mathbf{D}X(t) \frac{dW(t)}{dt}$$

Integrate with respect to t , we get

$$dX(t) = \mathbf{C}(t)X(t)dt + \mathbf{D}X(t)dW(t)$$

Suppose this stochastic differential equation is solved and for easier derivatives, it is expressed in a simpler form as follows:

$$dS = \mathbf{C}(t)Sdt + \mathbf{D}SdW(t).$$

By using Ito Lemma,

$$dV = \frac{dV}{dS}dS + \frac{1}{2} \frac{d^2V}{dS^2}dS^2 + \dots$$

$$dV = \frac{dV}{dS}(\mathbf{C}(t)Sdt + \mathbf{D}SdW(t)) + \frac{1}{2} \frac{d^2V}{dS^2}(\mathbf{C}(t)Sdt + \mathbf{D}SdW(t))^2 + \dots$$

By using $dX_1^2 = dX_2^2 = dt, dX_1dX_2 = \rho dt$,

omitting all insignificant terms, we have

$$dV = \frac{dV}{dS}(\mathbf{C}(t)Sdt + \mathbf{D}SdW(t)) + \frac{1}{2} \mathbf{D}^2 S^2 \frac{d^2V}{dS^2} (dW(t))^2$$

$$= \frac{dV}{dS}(\mathbf{C}(t)Sdt + \mathbf{D}SdW(t)) + \frac{1}{2} \mathbf{D}^2 S^2 \left(-\frac{1}{S^2} \right) dt$$

$$= \frac{1}{S}(\mathbf{C}(t)Sdt + \mathbf{D}SdW(t)) - \frac{1}{2} \mathbf{D}^2 dt$$

$$= \mathbf{C}(t)dt + \mathbf{D}dW(t) - \frac{1}{2} \mathbf{D}^2 dt$$

$$dV = \left(\mathbf{C}(t) - \frac{\mathbf{D}^2}{2} \right) dt + \mathbf{D}dW(t)$$

$$\int dV = \int \left(\mathbf{C}(t) - \frac{\mathbf{D}^2}{2} \right) dt + \int \mathbf{D}dW(t)$$

$$V(t) = V(0) \exp \left[\int_0^t \left(\mathbf{C}(S) - \frac{\mathbf{D}^2}{2} \right) dt + \int_0^t \mathbf{D}dW(S) \right] \tag{10}$$

The equation (10) can be expanded to capital provider's equity model and entrepreneur's equity model. The capital provider's equity *mudharabah* model is

$$E(t) = E(0) \exp \left[\left(\alpha(t)k - \frac{1}{2}(\lambda k)^2 \right) t + \lambda k(X(t) - X(0)) \right] \tag{11}$$

where $\alpha(t)$ is the drift of stock prices, λ is the volatility of stock prices and $(X(t) - X(0))$ are random values. The entrepreneur's equity *mudharabah* model is

$$Q(t) = Q(0) \exp \left[\left(\alpha(t)(1-k) - \frac{1}{2}(\lambda(1-k))^2 \right) t + \lambda(1-k)(X(t) - X(0)) \right] \tag{12}$$

Equations (11) and (12) can be used to forecast the return of investment for two parties in Bursa Malaysia.

RESULT AND DISCUSSION

This section presents the result and the discussion of the forecast investment and profit for capital provider and entrepreneur.

The equations (11) and (12) can be used to forecast the profit for two parties: the investor or capital provider and the entrepreneur. One of the examples is the forecast profit for *mudharabah* investment on AIRASIA. The result of the forecast profit for *mudharabah* investment on AIRASIA is shown in Table 2.

Table 2
 The Forecast Profit for *mudharabah* Investment on AIRASIA

Mudharabah Investment										
			Capital Provider				Entrepreneur		Single Investment	
Date	Time	Real Price (RM)	Forecasted Investment E(t)	Forecasted Profit	Forecasted Investment Q(t)	Forecasted Profit	Forecasted Investment	Forecasted Profit		
24-May-11	0.004	3.02	10239.02	239.02	10101.78	101.78	10343.1	343.1		
25-May-11	0.0079	3.04	10280.79	280.79	10119.46	119.46	10403.33	403.33		
26-May-11	0.0119	3.04	10322.73	322.73	10137.17	137.17	10463.92	463.92		
27-May-11	0.0159	3.01	10364.84	364.84	10154.91	154.91	10524.86	524.86		
30-May-11	0.0198	2.98	10407.12	407.12	10172.68	172.68	10586.15	586.15		
31-May-11	0.0238	3.02	10449.58	449.58	10190.48	190.48	10647.8	647.8		
1-Jun-11	0.0278	2.99	10492.21	492.21	10208.32	208.32	10709.81	709.81		
2-Jun-11	0.0317	3	10535.01	535.01	10226.18	226.18	10772.18	772.18		
3-Jun-11	0.0357	2.99	10577.99	577.99	10244.08	244.08	10834.92	834.92		
6-Jun-11	0.0397	3.05	10621.15	621.15	10262.01	262.01	10898.02	898.02		
7-Jun-11	0.0437	3.24	10664.48	664.48	10279.97	279.97	10961.49	961.49		
8-Jun-11	0.0476	3.2	10707.98	707.98	10297.96	297.96	11025.32	1025.32		
9-Jun-11	0.0516	3.2	10751.67	751.67	10315.98	315.98	11089.53	1089.53		

Table 2 shows that the forecast investment and profit for capital provider or investor, $E(t)$, the forecast investment and profit for the entrepreneur, $Q(t)$ and the forecast investment and profit for single investment which did not involve profit sharing rate. The initial capital is RM 10, 000 that is from capital provider and the profit-sharing rate is (70:30). From this table, it indicates that the investors and the entrepreneur are getting the maximum profit in the two week short-term investment. It means that if these two parties invest on the 23/05/2011,

they should sell the stock two weeks later on the 09/06/2011 since on that day the forecast profit for a capital provider and entrepreneur are RM 751.67 and RM 315.98 for AIRASIA.

CONCLUSION

The proposed stochastic *mudharabah* model is able to forecast the investment of two parties when the capital is invested in stock market. Profit is divided accordingly to the agreed predetermined rates between the two parties.

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